



COST-MINIMIZING TRAIN PATH MANAGEMENT: CURRENT AND PROPOSED CALCULATION METHOD

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Abstract – Serbian railway access charges for unused reserved capacity are being reformed. This paper develops a cost-minimization model for freight operators selecting train paths under both current (train-count-based) and proposed (train-km-based) penalty calculation method. Three long-term path types are analyzed — Regular (R), Regular on designated days (Rd), and Facultative (F) — together with ad hoc (A) path type. We derive closed-form crossover thresholds, prove that F paths dominate R paths in the deterministic scenario, and expose a systematic distortion in the current method for Rd paths: identical integer thresholds for different numbers of designated days produce disproportionate penalties. The proposed calculation method corrects this by penalizing proportionally to reserved train-km. A numerical example illustrates the magnitude of the distortion.

Keywords – train path allocation, access charges, cost minimisation, unused reserved capacity

1. INTRODUCTION

Infrastructure of Serbian railways (ISR) is transitioning to a new methodology for charging unused reserved train-path capacity. The current regulation penalises railway undertakings (RUs) for unrealised train departures at the origin of the allocated path, while the proposed calculation method, effective from the 2027 Network Statement, imposes penalties based on unrealised train-kilometres. Although the parameter values remain similar, the shift from a departure-count to a distance-based metric changes the economic calculus for every path category.

The change is especially significant for two scenarios common within rail Serbian freight operations: (i) trains that run only part of the reserved route (e.g., a loco path covering 60 % of the full route length), and (ii) Rd paths reserved for specific days of the week, whose occurrence count varies by month. The current regulation does not differentiate between these cases and full-route operations, whereas the new regulation does.

This paper formulates a unified cost-minimisation model, derives analytical solutions and threshold conditions for all four access strategies (R, Rd, F, A), and quantifies the distortion introduced by the old Rd penalty through a concrete numerical example.

2. PROBLEM DESCRIPTION

A freight operator plans capacity for a single route (length $L[\text{km}]$) over one month (d_m calendar days, $n_m = d_m/7$ weeks). Four different access strategies are available:

- R — Regular path: reserved every day, charged $d_m \cdot CP_i$ per month (where CP_i denotes requested path price).
- Rd — Regular on designated days: reserved only for d_m^d specified days (e.g., Tuesdays), charged $d_m^d \cdot CP_i$.
- F — Facultative path: reserved with weekly flexibility (any day), charged $n_m \cdot CP_i$.
- A — Ad hoc path: no advance reservation; surcharge $\delta \cdot CP_i$ per departure.

Here $CP_i = c \cdot L$ is the unit-route access charge (c [€/train-km]). Key parameters: $\alpha = 0.35$ (utilisation threshold for R and Rd), $\beta = 0.10$ (threshold for F), $\gamma = 0.20$ (penalty rate), $\delta = 0.10$ (ad hoc surcharge). The RU's total demand is V [train-km/month], deterministic.

3. MATHEMATICAL FORMULATION

3.1 Proposed model — train-km based penalties

Let $x^R, x^{Rd}, x^F \in \{0, 1\}$ indicate which path types are reserved, and $v^R, v^{Rd}, v^F \in \mathbb{R}^+$ denote realised train-km for each path type. Variable $q_0 \geq 0$ captures ad hoc

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train-km. Penalty thresholds: $ad_m L$ (R), $ad_m L$ (Rd), $\beta n_m L$ (F).

$$\begin{aligned} \min Z^{new} = & \gamma c \cdot \max(0, x^R ad_m L - v^R) \\ & + \gamma c \cdot \max(0, x^{Rd} ad_m L - v^{Rd}) \\ & + \gamma c \cdot \max(0, x^f \beta n_m L - v^f) \\ & + \delta c \cdot q_0 \end{aligned} \quad (1)$$

s.t.:

$$\begin{aligned} v^R + v^{Rd} + v^f + q_0 & \geq V \\ v^R & \leq x^R d_m L \\ v^{Rd} & \leq x^{Rd} d_m L \\ v^f & \leq x^f d_m L \end{aligned}$$

3.2 Current model — train based penalties

Let q^R , q^{Rd} , $q^f \in \mathbb{Z}^+$ be departure counts. Integer thresholds: $\tau^R = \lceil ad_m \rceil = 11$, $\tau^{Rd} = \lceil ad_m \rceil$, $\tau^f = \lceil \beta n_m \rceil = 1$.

$$\begin{aligned} \min Z^{old} = & \gamma CP_1 \cdot \max(0, x^R \tau^R - q^R) \\ & + \gamma CP_1 \cdot \max(0, x^{Rd} \tau^{Rd} - q^{Rd}) \\ & + \gamma CP_1 \cdot \max(0, x^f \tau^f - q^f) \\ & + \delta CP_1 \cdot q_0 \end{aligned} \quad (2)$$

s.t.:

$$\begin{aligned} q^R + q^{Rd} + q^f + q_0 & \geq V/L \\ q^R & \leq x^R d_m \\ q^{Rd} & \leq x^{Rd} d_m \\ q^f & \leq x^f d_m \end{aligned}$$

The key structural difference: the proposed model captures partial-route operation through continuous train-km (a train covering $K < L$ km contributes only K , not L). The current model counts departures regardless of distance run, creating the distortions analysed in Section 4.

4. ANALYTICAL RESULTS

4.1 R and F path optimality

Greedy allocation lemma: it is always cost-optimal to fill long-term path capacity before resorting to ad hoc access, in both models. Equating the penalty under one R path with the total ad hoc cost for Q full-route trains gives the crossover threshold:

$$Q_a^R = \lceil \gamma \alpha / (\gamma + \delta) \rceil \cdot d_m \approx 7 \text{ trains/month} \quad (3)$$

Below Q_a^R , ad hoc access costs less than carrying the R -path penalty. The current-model crossover uses $\tau^R = \lceil ad_m \rceil = 11$ instead of $ad_m = 10.5$, giving $Q_a^{old} \approx 7.33$ — slightly higher due to ceiling rounding. For F paths, the threshold is $\beta n_m L \approx 0.43L$ train-km (for proposed calculating method) or $\tau^f = 1$ departure (according to current calculation method): a single entire-route departure eliminates the F -path penalty in both models. Since F costs $n_m \cdot CP_1$ against $d_m \cdot CP_1$ for R (and $n_m < d_m$), F paths dominate R in the deterministic scenario for all $\beta < 7/d_m \approx 0.23$.

Figure 1 shows that with $Q = 11$ trains, even a 1% shortfall in route completion immediately generates a penalty with proposed calculation method, while implementing current method charges nothing.

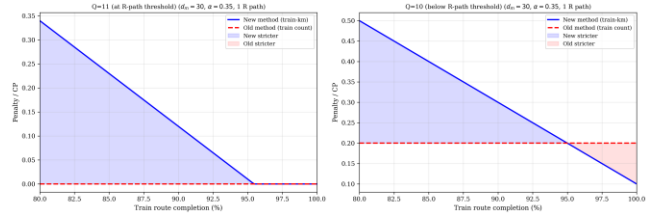


Figure 1. Partial-route penalty for $Q = 11$ (left) and $Q = 10$ (right), R path ($d_m = 30$, $\alpha = 0.35$). Proposed calc method (blue) is continuous; current method (red dashed) is a step function.

The divergence is larger below the threshold (right panel), as the current method gives a fixed step penalty, while the proposed one grades it smoothly with actual km shortfall.

4.2 Rd paths — cost boundary and distortion

For a Rd path with d_m^d designated days, the monthly reservation cost is $d_m^d \cdot CP_1$, vs. $n_m \cdot CP_1$ for F path type. The two are equal at the critical boundary:

$$d^{*d}_m = n_m = d_m / 7 \quad (4)$$

Below d^{*d}_m , the Rd path is cheaper than F but offers slots only on specified days. Above d^{*d}_m , F is both cheaper and more flexible. For January ($d_m = 31$, $n_m \approx 4.43$): Rd within months with five Tuesdays costs more than F ; Rd within months with three or four Tuesdays costs less than F . The ad hoc crossover for Rd paths in the proposed calculation method equals:

$$Q^{RdR}_a = \lceil \gamma \alpha / (\gamma + \delta) \rceil \cdot d_m^d = \rho_a^R \cdot d_m^d \quad (5)$$

This scales continuously with d_m^d . In the current calculation method $Q^{RdR}_{old} = \gamma \tau^{Rd} / (\gamma + \delta)$ scales only with $\tau^{Rd} = \lceil ad_m^d \rceil$, which equals 2 for all $d_m^d \in \{3, 4, 5\}$ for a threshold $\alpha = 0.35$. The current calculation method therefore assigns the same crossover threshold and the same penalties to three operationally different Rd paths. This is the distortion the proposed regulation corrects.

4.3 Numerical example

January ($d_m = 31$) may have 4 or 5 Tuesdays. A RU with one Rd path ("Tuesdays only") and Q complete departures faces the following costs. Penalty thresholds: according the proposed method — $1.40L$ (for 4 Tuesdays month) and $1.75L$ (for 5 Tuesdays month), while current method gives $\tau^{Rd} = 2$ in both cases.

Table 1. Rd penalties: January, 4 vs. 5 Tuesdays ($d_m = 31$, $\alpha = 0.35$, $\gamma = 0.20$, $\delta = 0.10$) and fixed path price

Q	Z^{new} , 4 Tue	Z^{old} , 4 Tue	Z^{new} , 5 Tue	Z^{old} , 5 Tue
0	0.28 CP_1	0.40 CP_1	0.35 CP_1	0.40 CP_1
1	0.08 CP_1	0.20 CP_1	0.15 CP_1	0.20 CP_1
2	0	0	0	0
Ad hoc ($Q=1$)	—	0.10 CP_1	—	0.10 CP_1

Table 1 shows three key findings. First, the current calculation method is completely blind to the difference between 4 and 5 Tuesdays: at $Q=0$ it charges $0.40 CP_1$ in both cases; at $Q=1$ it charges

0.20 CP_i in both cases. The RU pays the same whether the month had fewer or more reserved slots. Second, the proposed calculation method at $Q=1$ charges 0.08 CP_i for 4 Tuesdays — less than the ad hoc surcharge of 0.10 CP_i , so the Rd reservation is cost-effective. For 5 Tuesdays the proposed penalty is 0.15 CP_i , exceeding ad hoc, so the rational choice shifts to not reserving. Third, $Q=2$ clears the threshold and gives zero penalty in all cases.

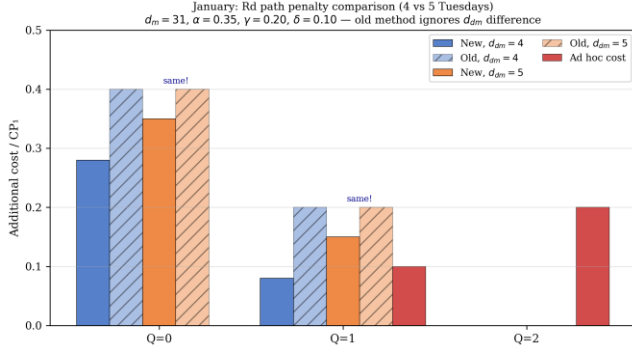


Figure 2. Rd penalty: 4 vs. 5 Tuesdays in January. Hatched bars (current method) are identical for both months at $Q = 0, 1$; solid bars (proposed method) correctly reflect the d_m difference.

5. COST COMPARISON AND SENSITIVITY

Figure 3 overlays the total additional cost against the number of trains Q for one path of each type R , Rd ($d_m^d = 5$), F , and ad hoc, under both calculation methods. Three demand zones are visible: an ad hoc zone (Q below the Rd crossover), a zone where reserving an Rd path reduces cost, and a high-demand zone where F achieves zero penalty at $Q \geq 1$. The right panel shows difference in calculated penalties, implementing two calculation methods, $\Delta Z = Z^{old} - Z^{new}$ for the R path and two Rd variants: the current method consistently charges more in the shortfall range, and the gap is proportionally largest for small d_m^d .

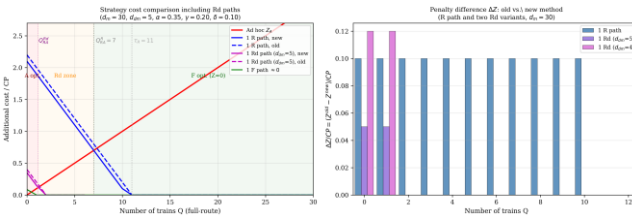


Figure 3. Left: cost curves for R , Rd ($d_m^d=5$), F , A under both calculation methods ($d_m=30$, $\alpha=0.35$, $\gamma=0.20$, $\delta=0.10$). Right: $\Delta Z = Z^{old} - Z^{new}$ for R and two Rd variants.

Figure 4 provides a unified sensitivity analysis: both panels plot the ad hoc crossover threshold vs. penalty rate γ for three values of α , comparing proposed (solid) and current (dashed) calculation methods. On the left side, R path ($d_m = 30$) is shown, while the right side shows Rd path (for the $d_m^d = 5$). For R paths, proposed and current crossovers track closely — the rounding of τ^R causes only a minor

upward shift in the current method curves. For Rd paths, all three dashed curves ($\alpha = 0.25, 0.35, 0.40$) collapse to a single line because $\lceil \alpha \cdot 5 \rceil = 2$ for all cases. The proposed method's solid curves' correctly diverge. There is no collapse for R paths, confirming that the distortion is specific to the integer-threshold treatment of Rd paths.

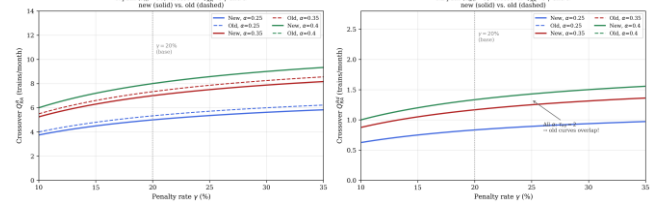


Figure 4. Sensitivity of crossover threshold for γ and α variations. Left: R path ($d_m=30$). Right: Rd path ($d_m^d=5$). All old-model Rd curves overlap ($\tau^{Rd} = 2$ for all α shown).

6. NETWORK EXTENSION

For a network of routes $i \in \mathcal{J}$ with different lengths L_i , charges c_i , and designated-day counts $d_{m,i}^d$, both models decompose into independent per-route subproblems, without shared capacity constraints. The network-level objective function for the proposed calculation method equals:

$$\begin{aligned} \min \sum_i [\gamma c_i \max(0, x_i^R, \alpha d_{m,i}^d L_i - v_i^{R,i}) \\ + \gamma c_i \max(0, x_i^{Rd}, \alpha d_{m,i}^d L_i - v_i^{Rd,i}) \\ + \gamma c_i \max(0, x_i^f, \beta n_m L_i - v_i^f) \\ + \delta c_i q_{0,i}] \end{aligned} \quad (6)$$

The Rd term carries the route-specific $d_{m,i}^d$, making the per-route distortion analysis of Section 4 directly applicable to each network arc. An identical decomposition holds for the current calculating method with $\tau^{Rd,i} = \lceil \alpha d_{m,i}^d \rceil$. Because per-route costs are separable, the global optimum is the union of per-route optimal strategies, enabling efficient parallel optimisation over an entire network.

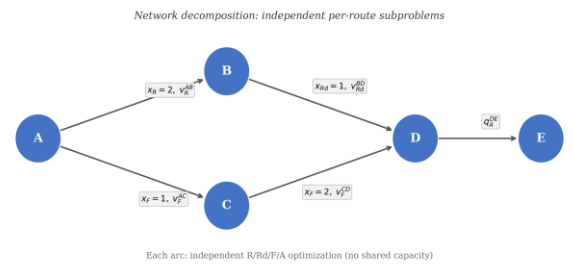


Figure 5. Network decomposition: each arc solved independently.

7. CONCLUSIONS

A cost-minimisation model for train path selection under current and proposed penalty calculation methods for unused reserved capacity has been formulated and solved analytically. Key findings are:

- F paths are optimal in the deterministic scenario: one departure per month eliminates the F -path

penalty, and F is cheaper to reserve than R for all $\beta < 7/d_m$, satisfied by all current Serbian parameter values.

- The R -path crossover $Q_a^R \approx 7$ trains/month (for months with $d_m=30$ days) marks the demand level below which ad hoc access costs less than reserving a R path.
- Rd paths have a critical cost boundary $d_m^{*d} = n_m$: above it, F is both cheaper and more flexible, making the Rd reservation economically unjustifiable.
- The current calculation methods systematically distorts Rd penalties: $d_m^d \in \{3,4,5\}$ always yield $\tau^{Rd} = 2$ and identical penalties, irrespective of the actual number of designated days reserved. The distortion ratio reaches $20\times$ for $d_m^d = 3$.
- The January example quantifies the impact: for $Q = 1$ train, the proposed calculation method correctly differentiates between months, charging $0.08 CP_i$ for the month with 4 Tuesdays (below the ad hoc cost of $0.10 CP_i$, so reserving is worthwhile) and $0.15 CP_i$ for the month with 5 Tuesdays (above ad hoc, so reserving is not worthwhile). The current calculation method charges $0.20 CP_i$ in both cases, failing to distinguish the two situations and

incorrectly signalling that the Rd reservation is never cost-effective relative to ad hoc access.

- The network model decomposes per route, enabling efficient operator's parallel optimisation over entire network.

Future work should extend the model to stochastic operator's transport demand, multi-month reservation windows, and competitive equilibria between multiple carriers sharing infrastructure capacity.

REFERENCES

- [1] Infrastruktura železnice Srbije, Izjava o mreži 2026 (Network Statement 2026), Belgrade, 2025.
- [2] European Commission, Directive 2012/34/EU establishing a single European railway area, Official Journal of the EU, L 343, pp. 32–77, Dec. 2012.
- [3] Borndörfer, R., Grötschel, M., Pfetsch, M.E., "A column-generation approach to line planning in public transport", *Transportation Science*, vol. 41, no. 1, pp. 123–132, 2007.
- [4] Caimi, G., Lübbecke, M., Möhring, R., Schaer, M., "A model for the optimization of the train scheduling problem", *J. Rail Transport Planning & Management*, vol. 1, no. 1, pp. 28–44, 2011.