



APPLICATION OF MATHEMATICAL MODELS IN RAILWAY WASTE MANAGEMENT SYSTEMS

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Abstract – The development of railway transport has contributed to an increase in the volume and dispersion of travel, which has also led to certain negative consequences, with the generation of significant amounts of waste becoming an increasingly important issue. However, the utility value of waste can be significantly enhanced through a well-organized system of collection, sorting, storage, and recycling, with waste playing an increasingly important role in the raw materials market used for energy recovery. The specific objective of this research is to improve the theoretical-methodological and professional foundations for optimizing the waste management system and to develop a mathematical model based on the principles of integer linear programming, aimed at optimizing the transport process and reducing negative environmental impacts. The model will be validated under real conditions on a selected section of the railway network (case study: the railway network in Vojvodina). The scientific contribution of the research is reflected in the advancement of methodologies for data systematization and the development of an applicable mathematical model for process optimization, through a detailed analysis of the causes of their occurrence, their intensity, and consequences.

Keywords – Waste management, railway, linear programming, multi-criteria analysis.

1. INTRODUCTION

The issue of waste generated in railway operations has long been relevant in Western Europe. In Serbia, however, it has not received sufficient attention due to low employee awareness, limited funding, cheap energy, inadequate legal regulations, and weak systematic control.

Waste management procedures include collection, on-site treatment, transport to recycling or disposal facilities, temporary storage options, and cost evaluation, along with establishing a documentation system.

To optimize these processes, this paper proposes a mathematical model that supports planning and design by enabling efficient waste flow management, reducing costs, minimizing environmental impact, improving resource use, strengthening recycling, and promoting a closed-loop material cycle.

2. ANALYSIS OF THE WASTE MANAGEMENT SYSTEM ON SRI AND EXAMPLES OF GOOD PRACTICE

Serbian Railways Infrastructure has adopted strategic documents promoting waste reduction, circular economy principles, and improved

environmental standards. However, in practice, challenges remain, including insufficient equipment, limited funding, and weak system organization.

The most common waste types include metal, concrete, wood, ballast, and municipal waste, while hazardous waste consists of oils, batteries, e-waste, and contaminated materials. Proper classification and separation are essential, as they determine further treatment and recycling options.

Waste flows involve generation, collection, transport, storage, and final treatment, supported by documentation and reporting systems. However, gaps exist in data continuity, along with cases of illegal or improper disposal.

Temporary storage is highly fragmented, with over 280 locations, many do not meet required standards. This approach is inefficient, leading to proposals for centralized facilities in Belgrade, Novi Sad, Niš, and Užice.

Overall, the system is not fully aligned with modern standards but can be improved through a systemic approach, infrastructure investment, and innovative solutions.

In Europe, modern railway waste management is based on sustainability and circular economy principles. German Railways (Deutsche Bahn)

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represent a highly developed system, with over 95% recycling, centralized management, and advanced IT tools enabling efficient monitoring and reuse of waste as secondary raw materials.

Croatian Railways represent a transitional system aligned with EU regulations, focusing on impact reduction and resource efficiency. However, challenges remain, including incomplete records, data inconsistencies, and underdeveloped infrastructure, particularly for hazardous waste.

3. DEFINITION OF THE MATHEMATICAL MODEL

The proposed model aims to optimize costs and determine the optimal spatial allocation of waste collection and primary sorting points, with the possibility of direct transport to final destinations.

It is based on integer linear and mixed-integer programming and includes: generated waste by type (hazardous and non-hazardous), stations that generate or collect waste, hub stations with storage capacities, and regional landfills as treatment facilities.

Two scenarios are defined:

I) Scenario 1 – waste from smaller stations is transported directly to local landfills;

II) Scenario 2 – waste from smaller stations can be transported via larger hub stations.

The objective is to determine the most efficient transport routes, minimizing costs and environmental impacts, particularly CO₂ emissions, as follows:

Let set I represent railway stations, i.e., waste generators; set J railway stations where waste is collected within the network (hubs); and set K waste treatment facilities, i.e., regional landfills (figure below):

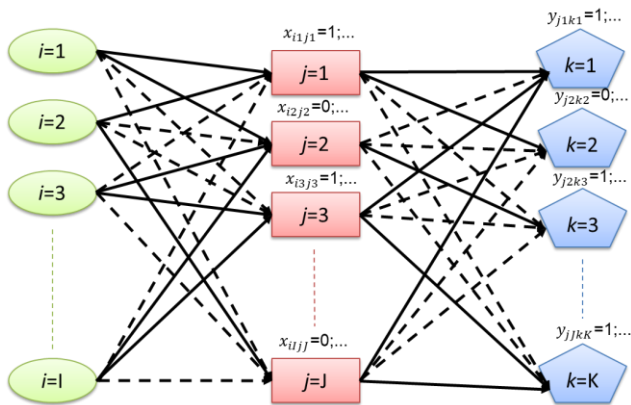


Fig.1. Representation of waste transport in the model

In addition, the set Q will represent the types of waste included in the model, i.e., hazardous and non-hazardous waste. From the above, it follows that the total quantity of waste of type q generated by station i is denoted as d_{iq} .

Transport costs for all relations “station – hub – facility” include the costs of waste collection and

transshipment. These aggregated and unified costs are determined separately for each type of waste and are denoted as:

- sfc_{ikq} – transport costs per ton of waste type q transported directly from station i to the disposal facility (regional landfill) k;
- sfc_{ijq} – transport costs per ton of waste type q transported from station i to the waste collection station (hub) j, including associated costs such as reception, storage, and necessary treatment within the hub j; and
- hfc_{jkq} – transport costs per ton of waste type q transported from the waste collection station (hub) j to the disposal facility (regional landfill) k.

The annual capacity for receiving and storing waste for each station j and waste type q is uniquely defined as the “capacity” of the waste collection station (hub) j and is denoted as A_{jq} . Similarly, the processing capacity of each waste treatment facility (regional landfill) k for individual waste types q is denoted as C_{kq} .

When referring to the fixed costs of opening and equipping waste collection stations (hubs) for waste type q, they are denoted as f_{jq}^{hub} , while for collection stations already in operation, these costs are proportionally lower. However, if the existing capacity of all stations is insufficient to accommodate the total generated amount of waste, it will be necessary to define, organize, and equip new waste collection stations (hubs). Therefore, f_{kq}^{fac} denotes the fixed costs of opening and equipping new waste collection stations (hubs) for waste type q.

Since the proposed model aims to answer the following key questions: (1) which stations and facilities are necessary for waste collection and treatment, and (2) how the collected waste should be distributed among stations, hubs, and regional landfills, decision variables and dependent variables are introduced into the model as follows:

u_{ikq} – the quantity of waste type q generated by station i (representing a portion of d_{iq}) that is transported directly from station i to the waste treatment facility k;

v_{ijq} – the quantity of waste type q generated by station i (representing a portion of d_{iq}) that is transported directly from station i to the waste collection station (hub) j;

w_{jkq} – the quantity of waste type q collected at the waste collection station (hub) j and transported from station j to the waste treatment facility k;

x_{jq} – a binary variable that takes the value 1 if the waste collection station (hub) j is “open” to receive waste type q, and 0 otherwise; and

y_{kq} – a binary variable that takes the value 1 if the

waste treatment facility (regional landfill) k is “open” to receive waste type q , and 0 otherwise.

In the case of Scenario 2, where it is allowed for waste generated at individual stations to be collected, stored, and consolidated at waste collection stations (hubs), the model is formulated as follows:

$$\begin{aligned} \text{minimize } F = & \sum_{i=1}^I \sum_{k=1}^K \sum_{q=1}^Q sf_{cikq} * u_{ikq} + \\ & \sum_{i=1}^I \sum_{j=1}^J \sum_{q=1}^Q sf_{cijq} * v_{ijq} + \\ & \sum_{j=1}^J \sum_{k=1}^K \sum_{q=1}^Q hf_{cj kq} * w_{jkq} + \\ & \sum_{j=1}^J \sum_{q=1}^Q f_{jq}^{hub} * x_{jq} + \\ & \sum_{k=1}^K \sum_{q=1}^Q f_{kq}^{fac} * y_{kq} \end{aligned} \quad (1)$$

where the constraints are defined as follows:

$$\sum_{j=1}^J \sum_{k=1}^K (u_{ikq} + v_{ijq}) = d_{iq}, \forall i \in I, \forall q \in Q; \quad (2)$$

$$\sum_{i=1}^I \sum_{q=1}^Q v_{ijq} = \sum_{j=1}^J \sum_{q=1}^Q w_{jkq}, \forall j \in J; \quad (3)$$

$$\sum_{i=1}^I v_{ijq} \leq A_{jq} * x_{jq}, \forall j \in J, \forall q \in Q; \quad (4)$$

$$\sum_{i=1}^I \sum_{j=1}^J (u_{ikq} + w_{jkq}) \leq C_{kq} * y_{kq}, \forall k \in K, \forall q \in Q; \quad (5)$$

For Scenario 1, where waste generated at stations is transported directly to waste treatment facilities, the model is simplified and formulated as follows:

$$\text{minimize } \Phi = \sum_{i=1}^I \sum_{k=1}^K \sum_{q=1}^Q u_{ikq} * sf_{cikq} + \sum_{k=1}^K \sum_{q=1}^Q f_{kq} * y_{kq} \quad (6)$$

The constraints ensure proper waste flow and capacity control in the model and require that all generated waste is fully collected from stations, while others ensures that no waste remains at collection hubs, meaning all collected waste must be transported to treatment facilities.

Finally, constraints also ensure that capacity limits at both hubs and treatment facilities are not exceeded, guaranteeing sufficient space for all waste.

4. VALIDATION OF THE PROPOSED MODEL AND DISCUSSION OF RESULTS

To verify the proposed mathematical model, a case study was conducted on the railway network of the Autonomous Province of Vojvodina in Serbia. The analysis covered 1,735 km of railway lines, 57 waste-generating stations, and existing and planned treatment facilities across an area of over 20,000 km².

Data for one representative month (October 2022) were used, with a total of 21,075 t of waste, including 16,367 t of non-hazardous and 4,708 t of hazardous waste. The model defines five hubs (Novi Sad, Pančevo, Ruma, Subotica, and Zrenjanin), with capacities based on current conditions and expert assessment, and includes detailed transport cost analysis.

Several scenarios were tested. The basic “do nothing” variant assumes direct transport to landfills without hubs, while other variants include hub-based systems with different facility combinations.

Results show that the non-optimized scenario generates the highest costs (EUR 1,087,003/month). The introduction of hubs significantly reduces costs to EUR 621,315–778,030, making these variants the focus of further analysis.

Transport flow analysis indicates that hubs greatly improve system organization, especially for non-hazardous waste (Figure 2), while hazardous waste is transported more directly due to strict safety and regulatory constraints (Figure 3).

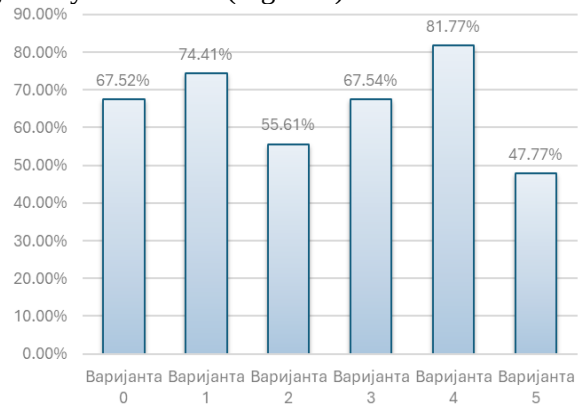


Fig.2. Quantity of non-hazardous waste treated via hubs

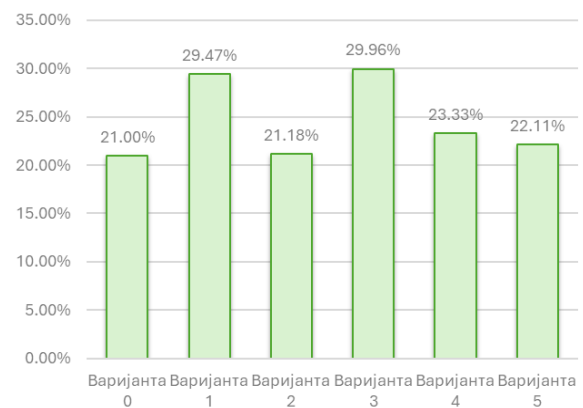


Fig.3. Quantity of hazardous waste treated via hubs

Transport work (ton-kilometers) is a key indicator affecting both total costs and environmental performance. Variant 2 achieves one of the lowest transport work values, while the least favorable variants show significantly higher levels.

CO₂ emissions analysis confirms that optimizing transport flows reduces environmental impacts, with

Variant 2 again ranking among the most favorable options.

Since no single variant is optimal across all criteria, a multi-criteria analysis was applied, considering financial, technical, environmental, social, and safety aspects. Using the TOPSIS method, Variant 2 achieved the highest utility value, which was further confirmed by the PROMETHEE II method, demonstrating the reliability and stability of the results.

Tab. 1. Result of the MCA using the TOPSIS method

	Var 0	Var 1	Var 2	Var 3	Var 4	Var 5
Result	0.482	0.625	0.688	0.464	0.545	0.529
Rang	5	2	1	6	3	4

Tab. 2. Result of the MCA using the PROMETHEE method

	Var 0	Var 1	Var 2	Var 3	Var 4	Var 5
Result	0.222	0.190	0.329	0.380	0.078	0.162
Rang	5	2	1	6	4	3

The synthesis of the obtained results shows that Variant 2 represents the optimal solution in terms of overall system efficiency. This variant ensures the lowest transport costs, reduced transport work, and favorable environmental performance, while maintaining adequate capacity utilization and meeting most technical and regulatory requirements. Compared to other variants, it achieves significant cost savings and a reduction in negative impacts, making it the most suitable option for practical implementation.

5. CONCLUSION AND FURTHER DEVELOPMENT

The developed mathematical model, based on linear integer programming, optimizes waste transport flows on the railway network to minimize costs and environmental impacts. The case study confirmed that introducing waste collection hubs and optimally selecting treatment facilities leads to financial savings, reduced transport and CO₂ emissions, and improved system efficiency. The model also proved to be universal and applicable across different network segments.

However, certain limitations were identified, mainly related to data availability and reliability, low digitalization, and the lack of a unified waste flow database, all of which affect analysis and decision-making.

Overall, the model represents a significant scientific and practical contribution to railway waste management, enabling cost and environmental optimization. Its further development requires a systemic approach, stronger institutional support, and greater environmental awareness among stakeholders as key conditions for sustainable railway development.

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